Chapter 1

Introduction

1.1 Background

The Global Positioning System (GPS) is a real time navigation system developed by the United States Department of Defense (DoD) and is used primarily for military purposes. GPS consists of a constellation of earth orbiting satellites that enable navigators on or near the surface of the earth to determine, **in real time**, their absolute positions and velocities to a few metres and decimetres per second respectively. The system is known both as NAVSTAR (for NAVigation System using Timing And Ranging) and more simply as GPS.

The system is funded and controlled by the US DoD but is partially available for civilian and foreign users. The accuracy that may be obtained from the system depends on the degree of access available to the user, the sophistication of the receiver hardware and data processing software, and the degree of mobility during signal reception.

Originally, the system was designed for twenty-four satellites, now more, placed four in each of six orbital planes, which in turn are evenly spaced around the equator. Each orbital plane is inclined to the equator by an angle of 55° and within each orbital plane, the four satellites were to be evenly spaced in almost circular orbits. The nominal radius of the orbit is to be 26,000 km or about 4 times the radius of the earth.

GPS is also used by surveyors to find position on the surface of the earth and can be used to find height. See, for example, Wells (1986), Leick (1990), Hofmann-Wellenhof et al (1992), Ackrody & Lorimer (1989). The height determined by GPS is height above the World Geodetic System 1984 (WGS84) ellipsoid, a mathematical model of the earth. Surveyors and engineers require orthometric height, which is height above the geoid, an equipotential surface very close to mean sea level. Defense Mapping Agency (1991) and, more recently, NIMA (2001) give details of the WGS84. The difference between these two height datums is termed *separation* and, although substantially constant in time, varies with location. Further data is required if separation is to be computed.

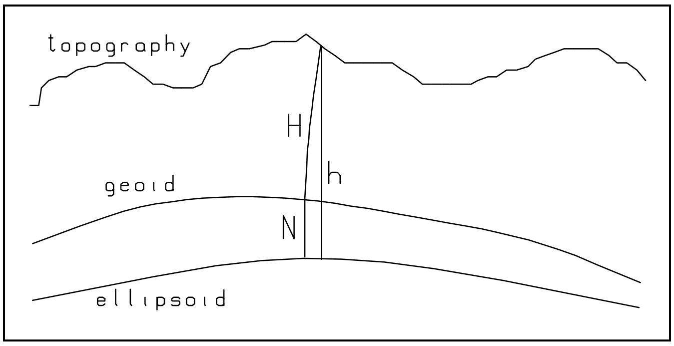

Figure 1.1 Orthometric height (H), Ellipsoidal height (h) and separation (N).

Knowledge of separation is essential for using GPS for the precise determination of orthometric height. The precise determination of separation over the area of a large project is a complex and costly operation. Technical solutions to this problem include:

Precise levelling coupled with GPS. Levelling is a very slow, land-based and labour-intensive operation, even with modern precise levels. The accuracy of such an approach is limited by the combined inaccuracies of GPS and levelling. Therefore such an approach is unlikely to ever be able to provide a geoid model of sufficient precision to be able to be used with GPS observations when precise orthometric height is required

Gravity observations. This requires theoretically world-wide, but practically nation-wide, gravity observations for the determination of separation at a single point. This has been done by the Ordnance Survey / Nottingham University. The theoretical basis for this work may be found in several texts such as Heiskanen & Moritz (1979), Vanicek & Krakiwsky (1982), Bomford (1980), Cross et al (1981), in order of theoretical rigor. The product is available from the Ordnance Survey, on a point-by-point basis, through their web site at Ordnance Survey (2001). A relative 2 - 5 cm geoid model, for distances up to 30 km is understood to be available for the whole of UK. However, GPS height differences, derived from baseline measurements may have standard errors in the order of 1 cm.

Gravity Satellite Missions. There have been a number of recent gravity satellite missions and more are planned. The GRACE mission with satellites launched in March 2002 consists of a pair of satellites about 220 km apart with nominal altitudes of 500 km. The satellite absolute and relative positions are found using on board GPS receivers and inter-satellite ranging. As the satellites pass through irregularities in the earth's gravity field. The distance between the satellites will vary and so by studying the changes in distance the form of the potential field and hence the geoid may be found. A gravity model will be generated every 30 days. A 180 degree and order spherical harmonic expansion of the earth's potential field smoothed to 160 degree and order will be computed. More details are at the GRACE website (GRACE, 2002)

The GOCE programme (GOCE, 2002) with a launch programmed in 2005 is planned to determine the rate of change of gravity between pairs of accelerometers with a sensitivity of about 4 milliEötvös in each of three axes. The unit of the Eötvös is the rate of change of gravity with distance. One Eötvös is defined as $10^{-5}(m/s^2)/10km$ which is $=10^{-9}s^{-2}$ (Units, 2002). It is hoped to determine the geoid to an accuracy of one centimetre.

Astrogeodetic levelling. Astronomic position is defined by the local gravity vector and may be determined by "Position Line" observations to the stars. Details of the method and the theoretical basis may be found in most geodesy textbooks such as Vanicek & Krakiwsky (1982), Bomford (1980), but in greatest detail in Robbins (1976). Geodetic position in WGS84, or more precisely in Europe, in ETRS89, may be found from GPS. The European Terrestrial Reference Frame 1989, ETRF89, is a reference frame that moves with the tectonic plate that Europe is on. ETRF89 is realised as the European Terrestrial Reference System 1989, ETRS89, through specific stations with defined co-ordinates. As such, the co-ordinates of all stations in ETRS89 do not change substantially. The difference between astronomic and geodetic position is termed the "deviation of the vertical" and is the same as the rate of change of separation in the direction in which deviation is stated. If the separation can be determined at a single point, such as by using GPS at a bench mark of known height, then the change in separation between the known point and the next point can be found by astrogeodetic levelling and so give the separation at the next point. In principle then, an astrogeodetic geoid model can be formed from the known separation at one point and deviation of the vertical at the same point and many others.

A relative astrogeodetic geoid may be produced at any chosen level of precision by increasing the number of points at which astronomic observations are carried out within a given area and by increasing the number of stars observed at each point. In short, to meet a range of specifications in terms of precision, relative astrogeodetic geoid models could be created. It is unlikely that the precision of the Ordnance Survey's gravimetric geoid can be improved without extensive further gravity observations and computations using height and density models.

A gravimetric geoid requires, theoretically, world-wide gravity data but practically, countrywide gravity data. Alternative, less extensive date will be required if a global spherical harmonic model of the earth's potential field is used to create a smoothed gravity model. The remove-restore technique can then be used to provide the fine detail in the local area for the geoid model. A relative astrogeodetic geoid only requires data within the area of interest and so is much easier and cheaper to develop if the astronomical observations can be reasonably cheaply obtained. At present that is doubtful and therefore it is a challenge for this research to find a more cost-effective way of doing so.

The UK's Ordnance Survey has recently stated new policy with respect to its use of reference frames and surfaces (Ordnance Survey, 2002). The Ordnance Datum Newlyn (ODN) is a height system realised by tidal observations at the South Pier at Newlyn in Cornwall and a Terrestrial Reference Frame (TRF) created by precise levelling between about 200 fundamental bench marks over mainland Britain and densified by over half a million lower-order bench marks.

The Ordnance Survey (OS) has a national GPS network that is designed to provide a three dimensional TRF to bring together ODN and the horizontal datum, OSGB36 by a transformation model. By using the National GPS Network points, GPS users will get co-ordinates of new points in the European Terrestrial Reference System 1989 (ETRS89) which can then be converted into OSGB36 co-ordinates and ODN heights

The Principal Triangulation of Britain was carried out between 1783 and 1853. Only one distance measure, the Houndslow baseline, was observed. The resulting network of primary control stations therefore had significant distortions, especially in scale at points some distant from the Houndslow baseline. The retriangulation associated with OSGB36 used the average of 11 of the original Principal Triangulation control stations in the definition of the new datum.

In the new definition of the OSGB36 TRF, the primary triangulation stations were taken as error free but the real distortions of OSGB36 are recognised in the ETRS89-OSGB36 transformation model. Within OSGB36, the standard error of horizontal position varies from nothing, by definition, for Primary control stations to 0.05m at 7km for "third order" control stations.

Height data published by the OS is orthometric height relative to ODN. A network of precise levelling lines was used to find orthometric height at other points across Britain. The OS states (Ordnance Survey, 2002) that "the accuracy of the precise levelling technique is rivalled by the combination of GPS ellipsoid heighting with a precise gravimetric Geoid model, which allows the ellipsoid height difference between two points to be easily converted to an orthometric height difference".

There is a vast resource of height data in Britain, about half a million bench marks. However many have not been levelled since the 1950s and especially in areas where mining has caused subsidence, such as around Nottingham, errors of several metres are known to exist.

In the ODN TRF, as with horizontal control, the “first order height control points” the fundamental bench marks, are considered error free. ODN orthometric heights are related to the to the GPS ellipsoid GRS80 through the OS National Geoid Model. Within the ODN TRF the standard error of height varies from nothing, by definition, for Fundamental bench marks to ±12mm for “third order” bench marks.

The OS plans to retain OSGB36 and ODN for mapping purposes but these will be available through co-ordinate transformations from ETRS89 to OSGB36 and ODN provided by the OS. The transformations are the National Grid Transformation OSTN97 and the National Geoid Model OSGM91. An advantage of this process is that height relative to ODN will be continuously accessible rather than just at discrete OS benchmarks.

The system of benchmarks in Britain has now been largely abandoned. In practice, surveyors will get height in the local area of interest by using GPS to transfer height from one or more of the OS active of passive GPS stations. For this to be successful the heights of those stations, especially the passive stations must be maintained with more rigour than was applied to the former system of benchmark. If not then the same kinds of subsidence and disturbance errors could affect the new system of passive GPS stations.

Soon, by definition the National Grid will become a transformation of ETRS89 and so can be directly related to the International Reference System, ITRS, and therefore to many other national geodetic datums. The OS state that “a real-time precise positioning service offering five-centimetre accuracy in ETRS89, OSGB36 and ODN co-ordinate systems is likely to become available from Ordnance Survey in the next few years”.

It must be hoped that height solutions will be rather better than the “five-centimetre accuracy” suggested in the previous paragraph. It is understood that OSGM91 has relative uncertainty of 0.01m up to 30km and rather more beyond. Clearly, error in height computed from GPS and the OSGM91 will reflect the errors in both the GPS derived co-ordinates and the geoid model used. The precision of height solutions from GPS will improve with advances in technology, software and the availability of future signals, especially the new L2C code and the third frequency, L5, due for implementation on new GPS satellites from 2003 and 2005 respectively. The approval and funding for the development phase Galileo (Galileo 2002) given on 26 March 2002 indicates that by 2008 there will be about 60 satellites, not counting Space Based Augmentation Systems or the Russian GLONASS, available for positioning. Receivers that can maximise the use of all available signals are likely to be able to compute position and height faster and more accurately than is available today. A geoid model capable of enabling height determination by satellite means, with the accuracy that may soon be achieved, will be required to support high precision engineering and scientific projects.

1.2 Aims of the investigation

The original aims of this investigation were:

a. To develop a method for the rapid determination of astronomical latitude and longitude including investigation of methods for optimising the selection of observational data and development of field procedures to minimise the time for the collection of observational data.

b. To investigate the application of rigorous statistical techniques to the precise determination of astronomical latitude and longitude.

c. To investigate the deviation of the vertical by astronomical observations at a chosen site.

d. To develop the application of mathematical techniques for the determination of a geoid model from astronomical deviation of the vertical data of a chosen area.

1.3 Structure of the Thesis

In Chapter 2 the context of the proposed work is reviewed and the methodology is stated in Chapter 3. Existing “ Position Lines” theory is reviewed and developed in Chapter 4 in a “least squares” context to take account of several previously un-modelled small errors. Observing and computing strategies are considered. Original contributions are made in terms of the development of the least squares approach to position lines in which refraction, vertical collimation and their rates of change are modelled as unknowns. Several possible solutions to the evaluation of a personal equation are examined and equations to correct for the effect on the observed vertical angle of an error in horizontal pointing are developed.

Chapter 5 concerns the construction and testing of suitable catalogues of stars and updating of co-ordinates to the time of observation.

In Chapter 6 observing and computing processes are examined in some detail to find a method that will give a good solution under the specific conditions of a particular set of astronomical observations. There is analysis of a method for detecting the instant of passage of a star across theodolite crosshairs using a photodiode. A method for linking GPS and video time by exposure of a GPS timed flash was developed and this lead to a video based method for detecting the instant of passage of a star across theodolite crosshairs.

The effect of lunar gravitation and barycentric centrifugal force on deviation of the vertical is examined in Chapter 7 and original formulae for the correction of observations of the vertical are derived.

The usual method of determining the topographic-isostatic effect is reviewed and a new simpler method that does not require a local terrain model is developed in Chapter 8. The model uses wedges across a longitudinal section on the line of greatest slope through the point under investigation.

Some theoretical and practical aspects are considered in Chapter 9. There is analysis of the Astrogeodetic Geoid Model using ideas based on Kaula’s rule of thumb and modelling the geoid by polynomial coefficients and by the interpolation of deviations. This leads to the application of “progressive nodes” as a means of avoiding the need for a full least squares solution.

A practical determination astronomical position is described in Chapter 10.

In the next Chapter, the relationship between the geoid and an ellipsoid is reviewed. Methods for the determination of the shape of the geoid are stated and compared. Historical progress in the determination of the geoid is reviewed, suggestions as to the future utility of the astrogeodetic geoid are made, and the place of astronomy in the determination of the geoid is reviewed.