

Chapter 11

Conclusions

11.1 Review of the original aims

The original aims of this study were stated in Chapter 1. As the study developed, so did its direction and therefore not all of the original aims have been addressed.

At the outset it was anticipated that it would be necessary to develop a prediction spreadsheet or software programme for manual video data capture, that is, with the video on a separate mount from the theodolite. Techniques for data capture by video methods would need to be developed. A database of star information that could easily be updated would need to be constructed.

Depending upon the quality of video images, it was anticipated that it would be appropriate to investigate methods for timing to better than 0.04s using characteristics of star-set/star-rise behind the theodolite graticule. Moreover a general least squares analysis would need to be used because the equation that connects the observations of time and the observations of the theodolite circle reading with the unknowns of latitude, longitude, parameters of the refraction model and theodolite vertical collimation contains two inseparable observations. Such a least squares solution would contain a full set of error statistics including the computation of residuals and their standard errors for the determination of outliers and for the true size of the position error ellipse. These are the statistical applications of quality assurance and quality assessment respectively. It was anticipated that such a solution would probably, subject to investigation, have a standard error of the order of $\pm 0".5 - 1"$.

It was expected that the process would require one hour for observations and two hours for computation per station, leading to a solution better than $\pm 1"$. Most of the two hours needed for data processing would be involved with abstracting time information from approximately 80 minutes of videotape.

With a video camera mounted on the theodolite, this would eliminate the need for lining up the camera with the theodolite for each star. In turn, it was expected that this would reduce the time per star to about 2 minutes, or 20 stars in 40 minutes.

It was anticipated that a motorised theodolite with video mount, driven by a laptop computer and software, would reduce the acquisition time per star to approximately 6 to 10 seconds per star or 20 stars in 3 minutes plus 15 minutes set up time. This would now be the same time requirement as that for the GPS data capture, a concurrent activity, and possibly similar to the travel time from point to point. Data processing time would also be reduced to about 1 hour per station because there would only be 3 minutes of videotape from which to extract time information.

Neither of these technologies were essential for this investigation but, had they been available, would have dramatically improved the numerical results of the investigation.

The original plan of work included:

- a. Design a computer based method for the prediction of star positions for any site and observation start time and hence create a site specific optimised observing programme.
- b. Use simulated observations, computed for the observing programme above, to investigate general least squares solutions for the determination of astronomical latitude and longitude.
- c. Conduct a short field campaign for the collection of observations, using the observing programme above. Compute astronomical latitudes and longitudes and review the quality and blunder detection statistics of the general least squares solution with a view to system improvement.
- d. Compare the solution derived by conventional graphical techniques with the least squares solution.
- e. Investigate how on-site solutions, rather than those which are post-processed, may be used to give integrity and quality indicators and to minimise on-site data capture time.
- f. Investigate the use of automation in the collection and processing of astronomical data. Technologies and techniques that, at the outset, that may have been available for investigation included:

Video for recording of star crossings.

Abstracting time information from video.

Time tagged theodolite data logging.

Servo/Robotic theodolites for rapid pointing.

- g. Investigate and select appropriate statistical methods for the determination of a geoid model from the deviation of the vertical information.

As work progressed, it became apparent that capturing images onto videotape was cumbersome and so an alternative method with a sensitive photodiode and with a video frame capture board was assessed.

In Chapter 1, the aim “to develop a method for the rapid determination of astronomical latitude and longitude including investigation of methods for optimising the selection of observational data and development of field procedures to minimise the time for the collection of observational data” has partly been met. The current version of the method developed by this author involves about two hours of data collection but rather more time spent on data processing per station. This is a significant improvement on traditional techniques that may involve one or two nights of observations and a day of computations. The major improvement is in the order of precision that can be achieved, improving from $\pm 3''$ to $\pm 0.2''$ in position. However, there is considerable scope for further improvement in the system, see Section 11.4.

The aim “to investigate the application of rigorous statistical techniques to the precise determination of astronomical latitude and longitude” has been reasonably successful. Least squares analysis by observation equations has been applied for the determination of lunar position, the determination of the time, in video frames, when a star crosses a crosshair and in the construction of geoid models. General least squares has been applied to find position with refraction, vertical collimation and their rates of change included in the parameters to be solved for.

The aim “to develop the application of mathematical techniques for the determination of a geoid model from astronomical deviation of the vertical data of a chosen area” has been attempted. Although several approaches to this problem were developed, which have application to a limited data set over a small area, there is insufficient rigour to justify application to substantial projects, such as those of a national scale.

11.2 Summary of Conclusions

Below is a summary of the main achievements and conclusions from each of the chapters.

In Chapter 4, Position Lines Theory, the least squares approach to position lines (in which refraction, vertical collimation and their rates of change are modelled as

unknowns) was developed and used in a spreadsheet to find astronomical position. Such an approach means that it not necessary to observe meteorological conditions on the ground and make the assumption that observations at one point can be used to model curvature of the optical path from ground to star.

The development of several possible solutions to the evaluation of a personal equation have been considered but it was discovered that the best solution is where estimation of the time that a star passes a crosshair is made without human judgement.

An equation to correct for the effect of an error in horizontal pointing on the observed vertical angle was developed so that an observed near zenith vertical angle could be corrected for the small but systematic error induced by not observing with the star on the theodolite crosshairs.

In Chapter 5, Star Positions, suitable star catalogues were constructed so that the possibility of ambiguity between close stars was avoided and a spreadsheet to update star co-ordinates to the date and time of observations was written to ensure that star data truly referred to the epoch of observation.

A method for detecting the instant of passage of a star across theodolite crosshairs using a photodiode was developed in Chapter 6, Observing and computing processes. Unfortunately, the method could not be tested in practice because the photodiode used was unstable at operating temperatures but the method is now available should, either a more stable diode become available or, a practical means of cooling a diode in the field be developed.

A method for linking GPS and video time by exposure of a GPS timed flash was developed so that there was no "personal equation" effect upon the determination of the timing of star passage over the theodolite crosshairs.

A video based method for detecting the instant of passage of a star across the theodolite crosshairs was developed and implemented in a spreadsheet. This enabled the determination of passage at a sub-video frame epoch level.

The prediction programme was optimised for star balance using a combination of various parameters so that any systematic effect of errors inherent in an unbalanced data set would be minimised.

Productivity with varying levels of automation was considered. It was concluded that the best possible productivity would be that obtainable where theodolite angle data is

captured with a data logger and the theodolite is motorised and programmed to point to stars on command.

Video time was precisely linked to GPS time by using a data capture rate of one frame nominally every 124 ms, that is a data rate slightly different from 8 frames per second. Therefore a precise but practical method to find the instant that a star crosses a theodolite crosshair using a video camera mounted on a theodolite with GPS time input has been developed.

A practical observing and computing aide memoire was written for use in the “field” so that the fairly complex procedure can be completed in the field without omission.

Theory to describe the effect of barycentric lunar gravitation on deviation of the vertical was developed in Chapter 7, The Effect of Lunar Gravitation and Barycentric Centrifugal Force on Deviation of the Vertical. Formulae for the computation of a correction have been derived so that this source of systematic error can now be corrected for. Where astronomical data has been used in the past, derived values of position may need to be corrected.

In Chapter 8, The Topographic-Isostatic Effect, the application of the topographic-isostatic effect on astronomical position was investigated. Formulae for use with a digital elevation model were derived but were of such complexity that application in a simple spreadsheet was not possible.

An approximate “rule of thumb” is that it is prudent to create a longitudinal section along the line of greatest slope in length at least four times the height of the highest point along that section was deduced. It is tentatively suggested that this should give a topographic-isostatic correction to $\pm 10\% \pm 0''.1$ of its true value in topography similar to that found in the UK. Therefore, at this level of precision, the computation load is significantly reduced and the need for raw DEM data is now reduced to that “along the line of greatest slope in length at least four times the height of the highest point along that section”.

It was found that, to minimise errors in the computed value of topographic-isostatic deviation, the best observing points are where there is an even slope, i.e. zero rate of change of slope.

The application and formulae developed here, using wedges across the local line of greatest slope, although not as rigorous as the DEM approach was much simpler and will give results that are of sufficient precision in non-mountainous areas such as most of UK.

Kaula's rule of thumb was related to the Astrogeodetic Geoid Model in Chapter 9, The Astrogeodetic Geoid Model. The determination of geoid models using polynomial coefficients and "progressive nodes" was examined. It was concluded that the former is the more rigorous approach but computationally much more complex. The latter is likely to produce results that are almost as good, but it was found that the determination of quality statistics is more difficult.

In the practical determination of astronomical position described in Chapter 10, a position solution with a standard error ellipse major axis of $\pm 0''.17$ was obtained with 20 observations and it was deduced that 50 good observations, taking 100 minutes to observe with manual pointing of the theodolite, should lead to a solution at the $0''.1$ level. With the aid of a motorised theodolite, the time of observations would also be significantly reduced. However, local temporal systematic refraction effects might then become more significant.

11.3 Practical Implications

The practical implications of this study are as follows.

- Astronomical position can now be determined to about $0''.15$ (5 metres) with less than 2 hours worth of observations or, potentially, less than half an hour of observations using a motorised electronic theodolite.
- In turn, this can lead to astronomically derived deviation as an economic data source for the determination of a local precise geoid. This may find a practical application where precise height is required from GPS such as in large engineering projects, for example bridge or tunnel river crossings. Where the engineering project is long and linear, such as with pipeline, canal or river works, the local geoid model is only required in and near the area of the works. In this case, an astrogeodetic geoid will be much easier to construct than a gravitationally derived one because it will require fewer observations.
- How much the process can aid the determination of national or continental geoid models would require further study. However, since precise astronomical position can now be determined with relatively little effort, then astronomy would lend itself more to the provision of a control framework for gravitationally derived geoid models, local independent checks upon gravitationally derived geoid models or the inclusion of astronomical

observations in astro-gravitational geoid models and geoid models derived from mixed observations.

- The system could also have application where change is expected over time in the direction of the vertical at a point, such as in tectonic and volcanic studies. In such cases, astronomical data would give information about change independent from other data sources such as precise levelling, gravity or geophysical measurements.
- There are significant improvements to the usefulness of the system that can be made and they are described in Section 11.4 below.

11.4 Further studies

If a geoid model is to be used with GPS to find orthometric height differences, then the quality of the model must be better than the height difference determined by GPS. With the current and future anticipated improvements in GPS capabilities, the requirement for a millimetre relative geoid will soon arise. For such a geoid to be determined by astrogeodetic means, there will need to be significant improvements in several aspects of the hardware and the data processing techniques.

- The quality of star co-ordinates available from current almanacs is probably not the weakest link in the process. However, any improvement will help, especially in the removal of systematic errors in star co-ordinates.
- Refraction is probably the area that could benefit most from further research. If the observations are made in a short period then there will be less likelihood of the refraction effects being cancelled out than with observations taken over a protracted period of time. Two approaches are possible.
 - Modelling systematic refraction effects at a given epoch and position due to known meteorological conditions such as the passage of a weather front.
 - The other is modelling total refraction effects, for example by “2 colour” methods, for the integrated meteorological effect. 2-colour solutions for meteorological effects have been used in EDM instruments such as the Terrameter for the measurement of distance. Observations on 2 frequencies are used with GPS. The problem with such an approach for direction measurement is that the difference in apparent direction of incoming blue and red light is that it will be about 2% of the total refraction effect and therefore to compute the refraction effect to 1” would require measurement of the

difference in altitude of red and blue light from the star to $0''.02$ - a formidable problem.

- Improved DEMs would aid the computation of the topographic-isostatic effect. Technically this would not be difficult if suitable map data already exists but the time and cost to produce the data sets could be significant. However, the refinement would need to be accompanied by improved density models for the computation of the topographic-isostatic effect. Figure 2 of Heiland et al (1998) suggests that the topographic-isostatic effect could be of several arc seconds in Austria. Marti (1998) suggests that the density of the topography in Switzerland can vary from $+0.4 \text{ g cm}^{-3}$ to -0.8 g cm^{-3} from the assumed value of 2.67 g cm^{-3} . Taking these two facts together suggests that there could be errors approaching $1''$ in the computation of the topographic-isostatic effect due to errors in the assumed topographic density.
- Further investigation is required to confirm that the method of finding topographic-isostatic deviation corrections by modelling the topography by infinitely long wedges perpendicular to the line of greatest slope gives a solution to $\pm 10\%$ of its true value in topography similar to that found in the UK.
- Improve the model for the effect of barycentric lunar gravitation on deviation of the vertical to take account of the fact that the lunar orbit is not circular and therefore lunar gravitation varies. With an elliptical orbit, the earth will accelerate and decelerate during its orbit around the barycentre further complicating the model.
- Improvements to the fundamental observations of vertical angle with electronic theodolites are desirable.